Dynamic Supply Chains with Endogenous Dispositions

James Paine
jpaine@mit.edu

Abstract

The movement of goods through a supply chain depends not only on the physical flow of goods but also on the economic decisions of each entity along the chain, including price discovery and inventory disposition decisions. This paper presents a methodological contribution to the System Dynamics community by developing a novel framework for supply chain models featuring economic decisions by combining three classic modeling methods: co-flow differential equation structures, spot price discovery, and multinomial logistic choice modeling. The relative economic values of possible dispositions of goods, including outright disposal, are considered. For work-in-progress, development is considered in terms of the economic value that an additional unit of time will bring to the finished good, and the interplay of these considerations drive goods through, or out of, supply chains. Incorporating these mechanisms can produce materially different behavior modes and can be applied to multiple levels of aggregation within a production process.
Introduction

Supply chain models have been a staple of System Dynamics modeling since Jay Forrester first developed the tools and methods to describe industrial processes and business cycles in the 1950’s and 1960 (Forrester, 1961). More recently, there has been a renewed interest in incorporating features of human decision makers in supply chain models, and in doing so better define a concept of Behavioral Operations Management (R. Croson et al., 2013; Gino & Pisano, 2008; Hämäläinen et al., 2013).

Though these calls for more behaviorally grounded approaches within supply chain research has emerged relatively recently, System Dynamics has had a behaviorally grounded approach from its inception. This observation has not been lost on researchers, and there have been recent calls to better integrate concepts, tools and frameworks between the OM and SD domains (Ghaffarzadegan & Larson, 2018; Größler et al., 2008; Morrison & Oliva, 2018; J. Sterman et al., 2015).

However, as discussed in the Literature Review section below, much of the supply chain modeling frameworks developed in prior System Dynamics, and even Operations Management, literature contain within them an implicit assumption that production is tied to production starts. However, for many contexts, the act of starting production does not necessarily guarantee that those units of inventory will become available to ship to customers. Aside from line losses (such as those due production errors, quality assurance sampling, or even natural losses such as crop failure or spoilage), each unit of inventory under development represents some measure of operational capacity that is reserved and thus prevented from other uses. Thus, the act of producing a unit of inventory contains within it an opportunity cost in the form of captured production capacity or other resource utilization.

This is not just a theoretical consideration, but one recently observed. The sudden emergence of the novel coronavirus commonly known as COVID-19 on the global stage in the early part of 2020 placed immense strain on supply chains and consumers. Articles from the middle part of that year describe “dumped milk, smashed eggs, plowed vegetables” as producers made the decision to terminate work in progress rather than move goods into a finished state for sale (e.g. Bauer, 2020; Corkery et al., 2020; Corkery & Taffé-Bellany, 2020; Yaffé-Bellany & Corkery, 2020; Zhou, 2020). Stated bluntly in one article “[Farmers] are being forced to destroy…fresh food that they can no longer sell” (Yaffé-Bellany & Corkery, 2020).

This article helps close this methodological gap by extending traditional inventory management and supply chain models found in System Dynamics literature by allowing for the endogenous
determination of dispositions of inventory and production in a supply chain via the application of multinomial logistic (MNL) choice modeling.

This is achieved via economically motivated decision rules that tracks the relationship between the age, or development time, of goods under production and their corresponding market value. In doing so, the value of unit of production started and placed under development in a work-in-progress (WIP) state to the producer is considered in the wider context of the interplay of supply and demand (and resultant price setting). The value of continuing development of a unit of production is considered versus alternative disposition routes, including even purposeful disposal if relevant to production environment, and the movement of WIP into a finished goods (FG) state is done not because a specific period has elapsed but rather because of the underlying economic value of that decision versus other disposition options.

An example model is presented and developed in this work that illustrates how inclusion of these mechanistic features can yield fundamentally different short and long-run behavior modes in similarly parameterized systems, and furthermore that this methodological framework can be applied to multiple levels of disaggregation of a production or aging process. Ultimately, it is the hope of the author that this work provides a novel and useful framework for capturing the economic tradeoffs and decision processes that are being utilized by producers in real supply chains, decisions that materially affect the flow of goods and the quality of the output of those systems.

Literature Review

As stated in the introduction to this article, System Dynamics has a long history with incorporating models of the physical flow of goods through supply chains with the human elements that interact with those supply chains, starting with Jay Forrester’s original modeling of the interactions of labor scheduling with production planning at General Electric (Forrester, 1961, 1989). Further investigations have included inventory-workforce interactions (Mass, 1975), production scheduling and planning via MRP systems (J. D. W. Morecroft, 1983), and consideration of the supply chain in larger settings that can yield business and capital equipment purchase cycles (Anderson & Fine, 1999; J. Sterman & Mosekilde, 1993).

Stability of production, inventory, and information signals within supply chains has been especially of interest in prior System Dynamics literature, with extensive research placed on the origins of instability, often characterized by the bullwhip effect (Lee et al., 2004), specifically arising from behavioral heuristics or limitation when used in ordering decisions (J. D. Sterman, 1989a, 1989b), and how these systems can be stabilized either via observations on the cognitive features of the people making
decisions (Narayanan & Moritz, 2015), or specific modifications to the information structure of the system (Rachel Croson et al., 2014; Rachel Croson & Donohue, 2006).

Inventory management models appear in much of the above referenced literature, and are described in detail in multiple System Dynamics textbooks (for example, see Chapters 18 and 19 of (J. D. Sterman, 2000), Chapter 5 of (J. Morecroft, 2015), or other illustrative uses of similar model structures in articles such as in (Kampmann & Oliva, 2009)). These classic models utilize basic behavioral feedback, tied to a producer’s desires inventory coverage level, to adjust a stock of inventory based on a perceived demand signal from a consumer. These core inventory management model can be readily extended by considering the time between the act of starting production of inventory and the availability of that inventory (see Chapter 19 and Chapter 20 of (J. D. Sterman, 2000) for a detailed example of this extension), or by considering other endogeneity such as the influence of inventory availability on customer demand patterns (as in Gonçalves et al., 2005 or Forrester's Market Growth model as replicated in J. D. Morecroft, 1983). More generally, other work has described principles of dynamic systems such as adding a minor loop to oscillatory systems like those seen in these inventory management models (Graham, 1977).

A key and fundamental consideration of the inventory management structures described above, and of the System Dynamics modeling framework in general is the purposeful incorporation of the behavioral features and heuristics employed by the individuals interacting with physical and information systems. These are often captured via decision rules that attempt to capture how a model of a human decision maker in the larger system incorporates information and observation to make a choice of action. This concept of modeling choices is not unique to System Dynamics and features heavily in modern economics literature as well. Specifically, discrete choice models have emerged over the last few decades as a method of empirically modeling the probability of observing outcomes among a finite set (chapters 17 and 18 of Greene, 2018 provide an excellent mathematical foundation). For the scenario where choices are collectively exhaustive, mutually exclusive, finite in number, that have the feature of independence from irrelevant alternatives (IIA) then the multinomial logit model can be used (McFadden, 1973).

This multinomial logistic choice modeling framework has become the most widely used in econometric analysis primarily because the formula relating the utility of each choice to the probability of that choice is closed form and easily interpretable (Henserh et al., 2005; Train, 2009). While extensions of this choice model exist, such as the probit model which relax assumptions related to the independence of the choices (Train, 2009), in supply chain modeling the set of disposition choices for inventory after or during any one step of production often follow the assumption of being collectively exhaustive, mutually exclusive, and finite. However, the usage of this modeling framework in a supply chain context is
relatively limited, with perhaps a notable exception for its use in models of transportation applications (e.g. Aloulou, 2018; de Bok et al., 2018), especially in comparison with its near ubiquitous use in other settings such as marketing (Chandukala et al., 2007 provides a good overview of marketing applications).

Within System Dynamics literature, the use of discrete choice modeling frameworks is similarly sparse, though much of the underlying mathematical theory overlaps with parameter estimation tools such as method of simulated moments (Hosseinichimeh et al., 2016; Jalali et al., 2015; Train, 2009). Explicit use of this choice framework in System Dynamics modeling has followed more closely to marketing applications, determining expected market share of different options given perceived utility either in a consumer context (Keith et al., 2017; Rahmandad & Sibdari, 2012). In a single industrial context found by this author, seemingly more superficially similar to the supply chain context discussed in this article, the use of the multinomial logistic choice model is still ultimately framed in terms of relative market share of fuel options for running electricity plants (Moxnes, 1990). Those examples from compartmental aggregate models in System Dynamics literature do highlight an important feature discussed in more detail below, namely that the probabilistic nature of the multinomial choice model allows for discrete and mutually exclusive choices at an individual level to be expressed as proportional outcomes at an aggregate level.

**Adding Disposition Choice Formulations into Supply Chain Models**

Consider the well-known inventory management model seen in Figure 1, adapted from (J. D. Sterman, 2000). This model captures the delay between starting production and having inventory on hand for shipment to a customer with a delay formulation between Production Start Rate and Production Rate, resulting in an accumulation of inventory in the form of work in progress or WIP. When creating a model of this system, this delay can be as simple as a fixed pipeline delay or is often represented as a more complex third-order delay to capture some sense of multi-stage production.
Each unit of inventory under development represents some measure of operational capacity that is reserved and thus prevented from other uses. Thus, the act of producing a unit of inventory contains within it an opportunity cost in the form of captured production capacity. When applied to work-in-progress inventory, the opportunity cost concepts can be extended by noting that act of holding of inventory and value-added development processes that are assumed to occur in a WIP state are not costless. Aside from the more obvious direct development costs of literal work in process, holding inventory in a such a state could prevent another unit of production starts from entering development (if production capacity is finite, fixed, and full), or at minimum contains some measure of opportunity cost by virtue of simply taking up physical space that could otherwise be utilized for a totally unrelated purpose.

Given some unit of production starts, the time that unit is under development is ultimately a choice of the producer, not fixed a priori. As illustrated in the recent example that opened this article, there exists environments where work-in-progress can be terminated at any point and the unit under production either moved to a finished goods state or even disposed of.

The farmers described in the introduction to this article made the difficult decision to destroy their crops because ultimately it made economic sense to do so. The costs of harvesting, processes, and transporting their goods exceeded the value they would get from selling finished goods, and even exceeded the opportunity cost of leaving the goods in the field, either in terms of holding up productive
capacity or due to the loss of value from spoilage in the ground. The time that a unit of production is under development, and considered work-in-progress inventory, may also have a meaningful economic impact on the final value of the product at hand. Consider a piece of software under development, where the value of the final product may increase with increased development time, but with decreasing marginal returns. Or consider a crop that is to be planted and harvested, with a specific window of maturation time in which it could be sold at full value.

The ‘Manufacturing Cycle Time’ shown in Figure 1 is not fixed in this choice-centric view of production. It is not even an average of a distribution of times. Rather it is an explicit choice made by the producer based on the economic features of the landscape in which they operate.

From the point of view of a single producer, each possible disposition of a unit of production in a work-in-progress state is likely to be mutually exclusive (e.g., in a single period a farm cannot simultaneously destroy, harvest, and continue to cultivate a single unit of food). Under a model of a single producer with fully known and fixed values (or costs) associated with each disposition decision, this economic decision becomes a straight-forward assessment of the expected value of each disposition route (for example weighing the of the costs of shipping and storing food versus the costs of destroying it, offset by the value that would come from selling if it were sold).

However, for a larger model of a system of producers, then the multinomial logistic choice model introduced above becomes more appropriate, representing the probability of a producer choosing any of the disposition routes. For an aggregate compartmental model of many producers this probability is proportional to the total work-in-progress inventory that is delegated to each of the possible disposition routes. As stated above, the use of multinomial logistic choice model has become extremely prevalent in other fields that consider the aggregate outcomes of human decision makers such as econometrics and marketing. Some limitations of the use of this modeling construct is that it assumes that individuals making these disposition decisions are fully informed with stationary costs, or in other words are fully informed about the cost structure of the system in which they are embedded, and that there is no correlation among choices (McFadden, 1973; Train, 2009). These assumptions can be relaxed in part by applying alternative methods that allow for correlation, like probit or mixed logit models (Revelt & Train, 1998; Train, 2009), or modifications to allow for stochasticity of observable data (Marcus, 1991).

The assumption that the individuals know the costs associated with each disposition follows from the assumption that the model is capture those actors that control the routing to those dispositions. However, the use of the multinomial logistic choice modeling framework is more generally consistent with classic discrete-choice foundations and contains within it a realistic assumption of human ordering
behavior, namely that the proportion of goods relegated to any specific disposition route is proportional the relative benefit of any one of those routes. Thus while other specific frameworks are likely feasible for specific applications, we argue that the multinomial choice model generally applicable and follows the principles for modeling decision making (Morrison & Oliva, 2018).

If these disposition routes follow the limitations of multinomial choice modeling (collectively exhaustive, mutually exclusive, finite, and IIA), then for some relative economic value \( \pi_i \) for choice \( X_i \), the probability of choosing \( X_i \) is given by the expression below:

\[
P(X_i) = \frac{e^{\beta \pi_i}}{\sum_{j=1}^{N} e^{\beta \pi_j}}
\]  

(1)

In the above, \( \beta \) is the weight the producer places on the concept of economic value. Under a full logistic model that we could fit to observed data, this becomes a free parameter. Here, we have no observed data, but rather a conceptual model. Thus, to simplify the model overall, we can fix values of \( \beta \) to be the inverse of some reference price for the producer (e.g., the price at which a farm sells its goods under normal steady state conditions). We can do this in part because the output of the multinomial choice model ultimately depends only on the relative difference in utility of each option (Greene, 2018), and this has the further advantage of allowing the relative values of each choice, \( \pi_i \), to be expressed in terms of prices and monetary values, while allowing the expression above to properly reduce to a dimensionless probability.

\[
\beta_i = \frac{1}{\text{reference price}} \forall i
\]  

(2)

How a producer assigns the relative values of each of the choices is a matter of modeling freedom and should ideally be based in observations of how real producers value these choices. The advantage of the logistic model is that changing or updating the assumptions that form this value assessment only changes the relative value of each choice, and thus the relative proportion of the units under development delegated to each option, but not the underlying model.

Figure 2 shows visually how the above multinomial logistic choice model can be incorporated into the basic inventory management model first presented in Figure 1. Note that the actual fraction of work in progress that can continue development is not directly used in the model, rather it is implied via the use of the multinomial choice model and the relative values of each disposition route.
In Figure 2, the stock of work-in-progress inventory is divided into three sub-groups per unit time by expression (1): Goods that the producer would like to continue to develop (i.e. stay in the WIP stock), goods the producer would like to dispose of, and goods the producer would like to move into a finished goods state. The relative size of each of these cohorts is based on the relative economic value of each disposition. In this aggregate model of work-in-progress inventory, the producer is assumed to have an ideal manufacturing cycle time that yields maximum economic value and determines the expected needed production start time (this is relaxed in the sections below), and a single processing time for work-in-progress inventory designated for movement to the finished goods state and that marked for disposal. The producer may have different times for these two dispositions, but for the sake of compact presentation they are combined here.
Dynamic Valuation of Work in Progress

The model visually presented in Figure 2 assumes fixed economic values for each inventory disposition. However, as discussed earlier in this article, the time that a unit of production is under development may also have a meaningful economic impact on the final value of the product at hand, and thus the value of either holding or shipping inventory may change with a concept of time under development or age of the work-in-progress.

Adopting a co-flow structure typically used to keep track of continually time-accruing attributes (Hines, 2005; J. D. Sterman, 2000) can be used to keep track of a concept of average development time (or average age, or average maturation time, or any similar measure that is appropriate to the system under investigation). Figure 3 illustrates this extension, where a co-flow structure is used to ultimately produce a concept of the average age of work-in-progress inventory, which in turn is used to adjust the expected value of moving WIP into a state of finished goods for sale to a downstream wholesaler or other customer.
Figure 3. Extending the model to track development time

Note that in the above formulation, the ‘Average Age of New Production Starts’ may have a value of 0 units of time, or some other non-negative value. For example, when applied to employee experience in a firm, an employee may arrive with some pre-existing experience. However, in the context of a food producer planting crops it may be safe to assume that newly planted crops arrive with no pre-existing maturation. The model is flexible to allow for this assumption to be relaxed based on specific
circumstance (for example, buying partially matured nut trees or fully matured sows rather than starting from seeds or piglets).

Furthermore, while it may be generally safe to assume that the ‘Rate of Age Gain’ is constant and unitary (i.e., 1 weeks/week or 1 years/year or similar). The formulation itself does allow for some flexibility if, as an example, a fertilizer was applied to speed maturation, or a drought hit and slows maturation down.

Applying the Framework to a Model Supply Chain

The purpose of the methodology developed above is to provide a flexible framework for applying this concept of age-depending economic features affecting the disposition choices of producers. The section below illustrates the utility of this framework by combining it with spot price discovery and capacity management in a simple example supply chain model to illustrate the additional insights adding this combination of multinomial choice modeling and price-value relationships can yield.

While there may be multiple ways to construct the interplay of supply and demand that ultimately forms the spot price at each interface point of producers and consumers in a market illustrated, the example below utilizes inventory-sensitive spot pricing most often seen with commodity products (Chen et al., 2009; J. D. Sterman, 2000; Whelan & Forrester, 1996). The core of this economic model is two balancing loops across each entity in the supply chain, with spot pricing driving either demand or supply. Given the current spot price in the market, the expected gross margin of the producer is affected, which in turn affects capacity utilization and production starts in a balancing loop. Additionally, the spot price is fundamentally anchored to what the market expects it to be, and this introduces a reinforcing loop around the spot price and the expected prices. These loops, in the context of a general n-entity supply chain, are visually summarized in Figure 4.
Figure 4. Ordering and Price Setting is Nested in Larger Interconnected Supply Chain

This price setting system seen in Figure 4 is then combined with the multinomial logistic choice modeling framework shown in Figure 3 to create the central mechanistic contribution described in this paper. The exact definition of the relationship between development time and the fraction of the full price that the producer can extract is context specific. The concept of ‘Effect of WIP Age on Producer Price’ seen in Figure 3 is left purposefully ill-defined. This relationship is ultimately context-specific can vary depending on the product under development and how the market values that product as function of the development or maturation time.

As discussed above, this relationship could be increasing with increased development time, but with decreasing marginal returns as in the case of software development. Or it could have an asymmetric gaussian shape with an ideal development time but with diminishing returns on either side of that time. This could be further simplified as an asymmetric trapezoidal shape if the window in which the full value of the good is not a single period but rather a window. Furthermore, as an example of this relationship, consider a product like a commodity food under cultivation. For such a product, there is an ideal window of maturation time, or work-in-progress age utilizing the language of the frameworks presented above, at which the food can receive its full economic value. Outside of his window, the producer or farmer can expect less than full value or even no value at all. This trapezoidal relationship between development age and price can be operationalized via expression (3).
More general trapezoidal shapes are possible that do not necessarily have linear changes in value (for example see (Dorp & Kotz, 2003)) and may be more appropriate in specific contexts. The Appendix to this article presents several other functional forms that this relationship could take on as well.

The example supply chain model was started at steady state using the parameterization shown in the Appendix and model documentation that accompanies this article utilizing the trapezoidal age-value relationship presented immediately above. The exact parameters of the model were chosen to be semi-realistic, but ultimately, they are illustrative only and not the focus of the discussion below or the illustration of the new modeling structures combining co-flow differential equation structures, spot price discovery, and multinomial logistic choice modeling. The model was then exposed to an exogenous shock in the underlying consumer demand for goods, increasing 50% over the baseline value for a total of 40 weeks (from week 10 to week 50 in simulated time).

First, Figure 5 shows the demand and production pattern of the system without the multinomial choice model nor the age-value relationship. In this baseline scenario, the supply chain is modeled in a manner like prior work, with production starts increasing to match increased demand, and all work-in-progress inventory eventually moved into a finished goods state. Most importantly, this system as parameterized is relatively insensitive to the pulse in consumer demand, quickly returning to the prior level of production once the demand surge subsides.

\[
 f(t) = \begin{cases} 
 0 & t \leq a \\
 \frac{1}{b-a} t - \frac{a}{b-a} & a < t \leq b \\
 1 & b < t \leq c \\
 \frac{1}{c-d} t - \frac{d}{c-d} & c < t \leq d \\
 0 & t > d 
\end{cases}
\] (3)

Figure 5. Baseline Response to Pulse in Consumer Demand
Figure 6 shows the same system with the same parameterization but with work-in-progress inventory advancing through the system subject to the multinomial logistic choice model introduced in expressions (1) and (2) and as illustrated in Figure 2. Incorporation of the multinomial logistic choice model means that prior to the shock in demand the producers will be disposing of some fraction of their work in progress every period. In net this means that the producer must carry more WIP than they would otherwise to meet the same level of demand in steady state. As consumer demand increases, increasing spot prices, the relative value of moving goods into a finished goods state versus continuing development or disposing of those goods also increases. This results in the immediate drop in the disposal of units and a small but still present drop in the average age of goods under development. As production starts begins to catch up with this increase in demand, the demand surge subsides. The value relationships shift again, with the relative value of holding goods in the WIP state or outright disposing of goods becoming more attractive versus finishing production and holding finished goods. This results in both an increase in maturation times and surge in disposal of goods.

Finally, Figure 7 incorporates a trapezoidal relationship between the average age or development time for the WIP goods and the ability for the producer to extract the full spot price in the marketplace. Now, holding goods in a WIP state beyond the maximum age at full value, or attempting to move goods
into a finished goods state before the minimum age at full value reduces the price the producer can demand in the marketplace.

Figure 7. MNL with Age-Value Relationship Model

Perhaps most importantly beyond providing an example of the use of this modeling framework, the three scenarios illustrated above (baseline, with MNL choice modeling, and with MNL choice modeling an age-value relationships) settle on three distinctly different qualitative modes of long run behavior. This is illustrated in Figure 8 for the inventory in the work in progress stock, but similar patterns emerge for other key variables, including production starts and spot prices.

The baseline model settles quickly with negligible oscillatory behavior, while incorporating more behaviorally realistic choice modeling frameworks generates oscillatory, but damped, patterns in production starts and inventories. Adding the relationship between development age and the price extracted to the marketplace results in undamped oscillations over the long run.

While these specific outcomes are of course a function of the specific parameter choices utilized in this model, it illustrates that the outcome of the model can be materially different if one chooses not to incorporate these mechanisms.
Vintaging Chains versus Aggregate Stocks

A valid critique of the structure presented in Figure 3 is that the work in progress by the producer is treated as a single aggregate collection of units with an average effective age under production. This structure abstracts away from capturing the outflows from work-in-progress to either finished goods or to purposeful destruction of goods of specific ages and instead considers only a concept of average age of each of these dispositions. Additionally, the exact distribution of ages of material that is currently under work-in-progress is abstracted away with only a mean value known.

While this aggregate framework may be a valid model in some contexts, in others it may be important, or at least of significant interest, to know an estimate of the distribution of ages of the work-in-progress inventory and the relative volume each age cohort is contributing to the net dispositions. To capture these features, a vintaging chain can be applied to extend the modeling framework developed above. This specific trade-off between a fully aggregate model and a more subdivided series of connected models is by no means unique to the context of this paper and has been explored extensively in prior literature in System Dynamics. The original population sector of Limits to Growth (Meadows et al., 1972) was built after an analysis of three different levels of aggregation and the relative tradeoffs and benefits of disaggregation into more precise age cohorts (Bongaarts, 1973). On the extreme end, direct comparison of fully agent-based models with fully aggregate models have helped illustrate relative utility or interchangeability of these two approaches, notably in the context of stochastic environments (Rahmandad & Sterman, 2008). Specific interchangeability and tradeoffs of differing levels of aggregation and disaggregation with real datasets have been explored (Fallah-Fini et al., 2013) along with
investigations of the effects of cohort disaggregation to the point of mathematically continuous cohorting (Eberlein & Thompson, 2013).

While much of the work previously done does directly apply to the modeling framework developed in this article (most notably Eberlein & Thompson, 2013), the analysis below makes the influence of the age-value relationship on the degree of disaggregation explicit. In doing so, this section reinforces that the choice of disaggregation ultimately is a free parameter left to the modeler, which should be based on what is behaviorally and physically realistic for the system being measured.

Consider the vintaging chain below in Figure 9. Here work-in-progress is split into $N$ evenly spaced vintages, followed by a single end-of-life cohort. This end-of-life cohort follows the same structure as the aggregate framework described above.

The age for each numbered age cohort increases regularly along the chain, starting with some initial age of production starts (typically assumed to have a numeric value of 0 units of time). Production that ages beyond the final numbered cohort enters an end-of-life structure like the aging co-flow developed in the aggregate model above as illustrated in Figure 3. Using this structure, the age of each cohort is known, including the final aggregate end-of-life life cohort on average.
Figure 9. Core Vintaging Chain
Under the aggregate model framework, the value of the entire stock of work-in-progress inventory is determined by the choice of age-value relationship. In the vintaging framework it can be applied to each cohort of ages. As an example, using the trapezoidal age-value relationship described in (3), in the vintaging extension we would expect that for low ages, almost all the inventories would be held to extract future value. At middle ages corresponding to the maximum full value, we would expect most of the inventory to be moved into a finished goods state. Finally, at high ages and especially in the terminal stock seen in Figure 9, we would expect most goods to be purposefully disposed of.

The multinomial logistic choice model, as illustrated in Figure 3, is applied to each part of this vintaging chain (though only one subpart is presented above). As presented here, the function that describes the Effect of WIP Age on Producer Price is the same across each vintage, though a trivial extension of this framework would allow for flexible value functions to be applied along the length of the chain.

Note that the structure developed here assumes that the average time to change cohorts categories is uniform. This formulation also assumes that economic valuation for the decisions around production and disposal are the same regardless of age. This is done here for clarity of exposition, and a more general model would apply the structure of Figure 3 separately, with possibly its own separate cost constructs, to each entity in Figure 9. However, the age-value relationship is being applied across the entire chain (e.g. with younger cohorts having a different economic value for being held onto and being allowed to mature versus later older cohorts). As the multinomial logistic choice model ultimately depends on the relative size of valuations of each disposition, even having uniform costing for disposal and production but different realizations of the value of the expected value of production means that each cohort will experience different splits along each disposition route. Additionally, keeping this age-value relationship and associated costs consistent in its application along the chain allows for more direct comparison of the aggregate framework to the vintaging framework, as discussed in the next section.

**Interchangeability of the Vintaging and Aggregate Frameworks**

By comparing the structure for the vintaging framework with that illustrated in Figure 3 for the aggregated framework, one key difference is that the concept of Average Age of WIP Inventory in the aggregated model is lost in the vintaging model. However, the two frameworks may be used largely interchangeably depending on the application and degree of utility of knowing the distribution of ages of goods in each disposition flow. This fits with the prior literature in this space discussed at the beginning of this section.
Given the same number of production starts (or planting rate in the example of crop cultivation), and the same processing time between both frameworks, it is possible to find an equivalent reference price that equate the net flow rates of each disposition route between these two models. This reference price refers to the relative weight placed on the value of each disposition decision as seen in expressions (1) and (2). As an example, consider the scenario utilizing the trapezoidal age-value function described in (3), and as parameterized in Table 1 in below.

### Table 1. Parameterization of Comparison of Vintaging and Aggregate Frameworks

<table>
<thead>
<tr>
<th>Parameter Name</th>
<th>Description or Note</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production Starts</td>
<td>Fixed and the same between the aggregate and vintaging framework</td>
<td>100 units/week</td>
</tr>
<tr>
<td>A</td>
<td>Minimum age of any value</td>
<td>2 weeks</td>
</tr>
<tr>
<td>B</td>
<td>Minimum age of full value</td>
<td>4 weeks</td>
</tr>
<tr>
<td>C</td>
<td>Maximum age of full value</td>
<td>6 weeks</td>
</tr>
<tr>
<td>D</td>
<td>Maximum age of any value</td>
<td>8 weeks</td>
</tr>
<tr>
<td>Number of Vintage Age Groups</td>
<td>Number of evenly aged vintaging groups, plus one end-of-life group</td>
<td>10 groups</td>
</tr>
<tr>
<td>Average Time to Change Age Categories</td>
<td>Chosen such that material older than time D moves into the end-of-life group</td>
<td>0.8 weeks</td>
</tr>
<tr>
<td>Processing Time</td>
<td>Average time for the producer, under either framework, takes to move material either into a finished goods state or to purposefully destroy it</td>
<td>3 weeks</td>
</tr>
<tr>
<td>Vintaging Framework Reference Price</td>
<td>Reference price by which the choices in the logistic model are evaluated at each vintaging age group</td>
<td>$1/unit</td>
</tr>
<tr>
<td>Spot Price</td>
<td>The exogenous (and here fixed) price that the market will pay for a full valued unit of production</td>
<td>$1/unit</td>
</tr>
</tbody>
</table>

Using the parameters in Table 1, for a reference price of $1/unit for the vintaging framework, it is possible to achieve, in equilibrium, the same net flow of goods moved into finished goods inventory with the same average age and with the same net flow of purposeful destruction in the aggregate framework with a reference price of $1.30463/unit. It is important to note that this value depends on all the parameters enumerated above, including the number of age cohorts, their divisions, and the shape of the age-value relationship.

One of the primary motivations for using a vintaging structure is to gain some degree of additional insight on the distributions of ages in each disposition category in the framework and this is where the aggregate and vintaging frameworks most significantly differ conceptually. As conceptualized here, the age of the material that exits production and passed along the supply chain is of the same average age and thus the same average value to maximum similarities between these two frameworks. In
the aggregate model framework, the average age of all three inventory dispositions is the same. However, the distribution of material in the various vintaging stocks and the resulting distribution of ages will be a direct result of the age-value relationship employed and thus will likely not be the same across the three dispositions in that framework.

Figure 10 shows the distributions of ages in the vintaging framework parameterized per Table 1. In the equivalent aggregate model, the average age across all three dispositions is calculated to be approximately 4.61 weeks.

![Ages of WIP Stock](image1)

![Ages of Production Flow](image2)

![Ages of Destruction Flow](image3)

**Figure 10. Example of Distribution of Ages in the Vintaging Framework**
Additionally, while the equivalence is maintained in equilibrium, the two structures begin to diverge when the common production start rate is anything other than fixed and constant for the specific reference prices determined above. Figure 11 illustrates this divergence, which focuses just on the net flows of purposeful destruction for illustrative purposes. While the reference price found that generates equivalent flows in equilibrium only qualitatively matches other state, it should be noted that for each of the inputs explored above there exists a reference price that matches the two frameworks. For example, the reference price of $1.21432/unit matches the exponential growth case but causes divergences in behavior in an equilibrium state.

![Figure 11](image)

**Figure 11. Comparison of Net Rates of Destruction between Aggregate and Vintaging Models**

While this is one example, it demonstrates that these two frameworks can be used interchangeably and equivalently when the limitations above are carefully considered. Specifically, it is
most interchangeable in when the general flow rates of interest are known, and the net flow into finished goods and destroyed goods that is most important for downstream processes.

Ultimately it is a choice of modeling freedom, and the choice should be based on both the reality of the system under investigation and the sources of value that come from the tradeoffs between a parsimonious model and more complete knowledge of the distribution of ages of production under a work-in-progress state.

**Discussion**

This article presents a methodological contribution to the System Dynamics community that expands on how decision makers choose to move goods through development processes in a supply chain. By combining multinomial logistic choice modeling with a model economic processes of price discovery, a more complete understanding of the behavioral features that determine the physical flow of goods through a supply chain can be explored, rather than simply assuming fixed or even multi-order delays in processing times.

Using a simple model supply chain, this paper further illustrated how the modes of outcomes of otherwise identical systems be materially different if one chooses not to incorporate the mechanisms developed in this work.

Degree of aggregation or disaggregation of the stock of work in progress was shown to be largely a free parameter left to the modeler and should be based on what is behaviorally and physically realistic for the system being measured. Both a fully aggregate and vintaging equivalent utilize choice modeling and economic concepts to determine the flow rates of goods through a supply chain and provide additional insight on the age (and thus corresponding value) of the goods in each disposition.

While the aggregate framework provides an average age across all disposition choices in a specific production activity, the vintaging framework allows for some additional insight on the distribution of ages, at the cost of significant additional modeling complexity and additional degrees of freedom in model design. For either framework, the distribution of ages (even if just the mean in the case of the aggregate framework) of material under production, or passed along any specific disposition path, emerge from the choices of the individuals, rather than being imposed exogenously.

While the context of economically motivated decision making of a producer in a supply chain was used here, specifically focusing on a stock of work-in-progress goods, the methods and frameworks developed above could be applied to any member of the supply chain and are flexible enough to be
applied to entirely other situations. So long as each disposition can be enumerated, and the relative value of each disposition described, then the logistic choice framework can be applied to derive the probability of each of those dispositions.

As applied here, this probability was assumed to apply uniformly across many producers and thus were proportional to the total flowrates of each inventory disposition. Another possible extension of this work is applying this same framework to models of individuals operating in a supply chain and making mutually exclusive disposition decisions each period. Under such a model, the logistic choice framework would be utilized to again from the probability of an individual choosing a specific disposition route, but only one such choice being realized each period.

Even with these limitations, it is the hope of the author that this work contributes towards rigorous and behaviorally grounded modeling efforts in the future by providing a novel and useful framework for other supply chain modelers to better incorporate features of the decision-making processes of producers that materially affect both the flow of goods and the quality of goods

References


https://doi.org/10.1111/j.1937-5956.2009.01099.x

https://www.nytimes.com/2020/04/18/business/coronavirus-meat-slaughterhouses.html%0A


https://doi.org/10.1016/j.jom.2012.12.001

https://doi.org/10.1287/mnsc.1050.0436

https://doi.org/10.1111/j.1937-5956.2012.01422.x

https://doi.org/10.1016/j.trpro.2018.09.049

https://doi.org/10.1007/s001840200230

https://doi.org/10.1002/sdr.1497

https://doi.org/10.1002/sdr.1508


Appendix to Accompany the Article

Dynamic Supply Chains with Endogenous Dispositions

Table of Contents

Model Availability .................................................................................................................. 3
Formulation Details for the Methodological Comparison Model........................................... 8
  Defining the Market ........................................................................................................... 9
    Effect of Inventory Coverage on Price ............................................................................. 11
    Effect of Expected Gross Margin on Demand ................................................................. 11
    Effect of Spot Prices on Demand .................................................................................... 12
Production Starts and Capacity Management ...................................................................... 14
Discounting the Spot Price based on Development Age ..................................................... 15
Quantifying the Age-Value Relationship ............................................................................ 16
A Multinomial Logistic model of Crop Dispositions ............................................................ 17
Valuing WIP Dispositions ................................................................................................. 18
Valuing FG Inventory Dispositions .................................................................................... 20
Some Alternative Functional Forms for the Age-Value Relationship .................................. 22
Parameterization of the Methodological Comparison Model ............................................. 24
References to the Appendix ............................................................................................... 29
Table of Figures

Figure S1: Switching to ‘Old Sketch’ in Vensim 9.0 and later................................................................. 3
Figure S2: Example of the Dashboard View of the Methodology Comparison Model.................................. 4
Figure S3: Detail of Aggregate Framework Embedded in Full Model View of the Methodology
Comparison Model.................................................................................................................................... 4
Figure S4: Partial Example of the Dashboard View of the Framework Comparison Model in Vensim...... 5
Figure S5: Detail of Aggregate Framework View in the Framework Comparison Model............................ 6
Figure S6: Detail of Vintaging Framework View in the Framework Comparison Model............................. 7
Figure S7: Example of Viewing the Supporting .mdl File in Notepad on Windows................................. 8
Figure S8. Overview of Methodological Comparison Model...................................................................... 9
Figure S9. Core Two Balancing Loops Inventory-Based Spot Prices.......................................................... 10
Figure S10. Ordering and Price Setting is Nested in Larger Interconnected Supply Chain ....................... 10
Figure S11. Examples of the Formulation of Demand versus Expected Gross Margin .......................... 12
Figure S12. Examples of the Formulation of Demand versus Spot Price................................................. 13
Figure S13. Producer Capacity Utilization versus Expected Gross Margin.............................................. 15
Figure S14. Example of Trapezoidal Function Discounting the Value of Crops based on Maturation...... 16
Figure S15. Examples of Price-Value Relationships.................................................................................. 23
Figure S16. Detail on the Coflow Age Tracking Structure.......................................................................... 25

Table of Tables

Table S1. Parameterization for the Age-Value Relationship in the Methodological Comparison Model.. 24
Table S2. Parameterization for the Co-Flow Structure Monitoring Average WIP Age ........................... 26
Table S3. Parameterization for Time Constants ........................................................................................ 26
Table S4. Parameterization for the Effects of Inventory Coverage and Elasticities .............................. 27
Table S5. Parameterization for Producer Costing..................................................................................... 28
Model Availability

Accompanying the main article and this appendix are the full models available as .mdl files, along with supporting data files to illustrate the how specific analyses were run and figures generated. The .mdl files can be open and run using Vensim software, developed by Ventana Systems, Inc. A free version of the Vensim software for personal use, along with a standalone model viewer, is available from Ventana Systems, Inc.

The models were originally developed in Vensim version 8.2 and revised in version 9.1.1. As of Vensim version 9.0, the visual style of this software package has changed significantly. The .mdl files are still fully viewable in Vensim these later versions of the software, but layout and text changes may make viewing the model slightly more difficult. The author suggests, if using Vensim version 9.0 or later, to view the associated .mdl files in the ‘Traditional Sketch’ or ‘Old Sketch’ layout. In Vensim version 9.0 this can be toggled via the Tools menu as seen below. All screenshots of software menus or images depicting the model layout or menu screens in the Appendix were done in this ‘Traditional’ view scheme.

![Figure S1: Switching to ‘Old Sketch’ in Vensim 9.0 and later](image)

Ventana Systems, Inc provides detailed documentation on the Vensim software, including how to manipulate and examine specific formulations. However, the reader may quickly explore the influence of parameter choices on the model via the SyntheSim mode on the main Dashboard view of the model. This can be accessed by pressing the corresponding button in the top toolbar of the software as seen below:

![SyntheSim](image)
For the supporting model comparing the core methodological framework, the .mdl file is divided into views: an overview Dashboard, a view of the full model itself, and several views that detail specific reporting or supporting structures. Different views can be access via the buttons in each view, or via the view menu. Examples of these two views (but not the entirety of these views) are provided below.

**Figure S2: Example of the Dashboard View of the Methodology Comparison Model**

**Figure S3: Detail of Aggregate Framework Embedded in Full Model View of the Methodology Comparison Model**

As a note, this model view is presented in its entirely and largely has no hidden structure or hidden causal connections. The model is still provided here for the interested reader and allows for the reader to investigate in detail how the frameworks developed in the main article are practically applied as subcomponents in a larger model, along with alternative value-age relationships.
For the model illustrating the details of the aggregate and vintaging framework, the presentation is designed to allow for comparison of the outputs of both frameworks when subjected to the same inputs. The .mdl file is divided into several views, most notably an overview Dashboard, and detail views of each framework. Different views can be accessed via the buttons in each view, or via the view menu. Examples of these views (but not the entirety of these views) are provided below.

**Figure S4:** Partial Example of the Dashboard View of the Framework Comparison Model in Vensim

Note that the vintaging framework is presented for 10 age cohorts, and this is fixed by design. This is to make the presentation of the framework direct and easy to interpret without the need for subscripting or
array formulations. This presentation can be greatly simplified via array approaches but doing so hides the underlying interplay of choices in the vintaging structure. However, the cost of this choice is a highly cluttered display of the fully connected model, along with difficulty in adjusting the number of cohorts. This fully connected view is present in the .mdl Vensim file, but the author encourages readers to focus on the detailed view of the beginning and end of the illustrative vintaging chain for ease of understanding.

Figure S5: Detail of Aggregate Framework View in the Framework Comparison Model
Furthermore, the .mdl files provided may be opened in any program that is able to read UTF-8 encoding and the formulations directly viewed in plaintext. Examples of program that can open the .mdl file for direct viewing in plaintext include Notepad in the Windows operating system and Textpad in the Macintosh operating system. An example of this view of the model file is seen in Figure S7.
The models developed for this article are also fully documented utilizing the SDM-Doc tool described in (Martinez-Moyano, 2012). The output from this documentation tool is available alongside the .mdl files.

**Figure S7: Example of Viewing the Supporting .mdl File in Notepad on Windows**

Formulation Details for the Methodological Comparison Model

In the main article, a simplified supply chain model that allows for the switching on and off of methodologies is used as an illustrative example of the use of the frameworks developed in a larger model. The sections below provide additional detail on the development of that comparison model, focusing on details that are not necessary to illustrate the frameworks developed in the main article, but are still of interest in the dynamics in this overarching system. As a note, some portions of the explanatory text from the main article are repeated below where needed to create a self-contained description of this model.
The model consists of a producer who manages goods flowing through two stocks: Work in Progress and Finished Goods. The figure below provides a high-level visual overview of the model, with each subsequent section providing more operational detail.

**Figure S8. Overview of Methodological Comparison Model**

**Defining the Market**

While there may be multiple ways to construct the interplay of supply and demand that ultimately forms the spot price at each interface point in the market illustrated in Figure S8, the loops defined as B2 and R utilize inventory-sensitive spot pricing (Chen et al., 2009; Sterman, 2000; Whelan & Forrester, 1996). These effects also feedback into B1 and even affect the loops that determine the relative value of each inventory disposition formed in B3+4.

The core of this economic model is two balancing loops across each entity in the supply chain, with spot pricing driving either demand or supply.
However, the above entity may exist in a chain of upstream and downstream entities, each ordering from their suppliers and selling to their own customers. This effects the ‘Expected Gross Margin’ and introduces another balancing loop. Additionally, the spot price is fundamentally anchored to what the market expects it to be, and this introduces a reinforcing loop around the spot price and the expected prices. These two new loops, in the context of the larger supply chain, are seen below:

**Figure S10. Ordering and Price Setting is Nested in Larger Interconnected Supply Chain**
Effect of Inventory Coverage on Price

One of the key features of the pricing model visually summarized above is the effect of inventory coverage on pricing. In net, a model will capture the downward sloping relationship between additional inventory (beyond a set inventory coverage goal) and the price offered by the firm holding that inventory.

\[
\text{Effect of [Entity] Inventory Coverage on [Entity] Price} = [\text{Entity}]\text{Inventory Ratio}^{-\text{sensitivity}}
\]  

(1)

The sensitivity is a parameter that determines how much the price will raise or lower given a change in inventory coverage. As formulated here, sensitivity is assumed to be a positive value, with higher values corresponding to increasingly concave response functions.

Another feature explored in the above formulation is a ‘cap’ on the maximum multiplier that inventory coverage could have on price. I.e., if inventory coverage approaches 0 (there is no inventory to sell), then the effect on the price will approach infinity. This does not happen as increase in spot prices drives down demand from downstream customers, preventing the final marginal units of inventory from ever being sold in practice.

Effect of Expected Gross Margin on Demand

A concept of expected gross margin can be used to influence production in the case of the producer, and demand in the case of all other entities in the supply chain, with increases in expected Gross Margin assumed to induce greater production or demand.

There may be multiple methods of incorporating this relationship here, including truncated sigmoideal functions and directly applying table functions. For this example, consider a simple truncated linear representation that meets the following criteria:

1. Passes through the point of (1,1) on a normalized scale
2. Is truncated at an upper maximum multiple on demand
   a. This assumes that it is infeasible for an entity will ever request more than some multiple of its reference demand at any expected future profit level
   b. This could be due to several possible factors not explicitly modeled such as storage space constraints, transportation limitations, or risk of spoilage).
3. Is truncated at a lower level of demand
   a. In other words, it is bounded at a minimum acceptable gross margin, which could be greater than 0%
b. Paratactically this means the line passes through the point of \((\text{Minimum Normalized GM, 0})\)

Given points 1 and 3 above, the slope of the line is defined fully by the specification of the minimum acceptable gross margin at which any demand or production will exist, and the definitions of the reference gross margin and corresponding reference demands. Examples of what this curve looks like can be seen in Figure S11 below.

![Figure S11. Examples of the Formulation of Demand versus Expected Gross Margin](image)

It should be emphasized that this curve is based on the expected gross margin to influence demand, which is in turn can be based on smoothed perceptions of previous prices that the entity has experienced.

**Effect of Spot Prices on Demand**

The above effect on demand due to expected gross margin does have some element of sensitivity to cost built in from the definition of gross margin. However, it is purposely done in relationship an expected gross margin based on a smoothed view of previous prices (both costs for goods bought and the prices at which they were later sold).
To affect demand based on the instantaneous spot price experienced by each entity, consider a linearly decreasing relationship that captures decreasing demand with increasing prices, with the slope of that relationship affected by some elasticity of demand. The functional form of this expression is seen below:

Effect of Price on Demand

\[
\text{Effect of Price on Demand} = \text{MIN}(\text{Maximum Multiplier}, \text{MAX}(0.1 + \text{Demand Curve Slope} \\
\times \frac{\text{Price} - \text{Reference Price}}{\text{Reference Demand}}))
\]

Where:

\[
\text{Demand Curve Slope} = \frac{-\text{Reference Demand} \times \text{Reference Elasticity}}{\text{Reference Price}}
\]

An example of the shape of this function for various values of elasticity are seen below, where \( e \) refers to the Reference Elasticity in expression (3) above.

![Graph showing the effect of price on demand for various elasticity values](Image)

**Figure S12. Examples of the Formulation of Demand versus Spot Price**
Here I purposely use the spot price to determine the effect of this instantaneous demand. This is purposeful designed to be immediate, in contrast to the effect from expected gross margin which is based on a smoothed concept of both prices and costs.

Combined, the relationships described in the economic market for this commodity good in which the new modeling framework presented here can be applied.

**Production Starts and Capacity Management**

The producer considers two different conceptualizations of profitability: the incremental profitability of an additional unit of production (utilizing just the variable costs of production), and the expected profitability from expanding production capacity (utilizing a fully allocated cost of production).

As a note, in this example this relationship utilizes the ‘Producer Expected Price’ which is the spot price smoothed over a short time range. The producer considers the price relative to expected costs to form a gross margin estimation when making capacity change decisions. Here, this expected gross margin utilizes a *fully allocated* unit cost. This expected gross margin determines the effect on desired capacity.

As discussed in other System Dynamics models of commodity markets (notably Chapter 20 of (Sterman, 2000)) utilization is a function of expected gross margin. Furthermore, utilization of existing capacity is unlikely to be at 100% when averaged across all pieces of owned capacity unless at very high levels of expected profitability. The exact shape of this relationship will vary by industry and even by individual producer or individual piece of owned unit of production capacity. To qualitatively capture this behavior, consider a function which approximates a curve approaching the CDF of a collection of different land (capacity) at different utilization depending on local factors. One such curve, and the one utilized in this example is shown in Figure S13.
Figure S13. Producer Capacity Utilization versus Expected Gross Margin

Discounting the Spot Price based on Development Age

A central piece of the framework presented in the central paper is the relationship between the age of the goods being produced and the value they derive in the marketplace. For the example in the main article, consider a relationship of the same trapezoidal functional form as that described in expression (6) and illustrated in Figure S14. Furthermore, the example below utilizes the single aggregate stock of work-in-progress inventory instead of a more granular vintaging framework as described in the main article.

The quantification of the opportunity cost capturing the tradeoff between time that a unit of potential inventory spends under production (or development) versus the amount of economic value the producer can expect to get from its eventual sale is explored in more detail in the sections below.

\[
Effect \ of \ Age \ on \ Price = f(Average \ Age \ of \ WIP)
\]

(4)

\[
Producer \ Spot \ Price = \frac{Effect \ of \ Age \ on \ Price \ \times \ \text{Spot \ Price \ for \ Full \ Mature \ Goods}}{\text{Spot \ Price \ for \ Full \ Mature \ Goods}}
\]

(5)

In the above, the ‘Spot Price for Full Mature Goods’ is defined via the method described in expression (1), and is a function of the inventory coverage of the producer.
Quantifying the Age-Value Relationship

As discussed in more detail in the main article, this relationship that defines ‘Effect of Age on Price’ is context-specific can vary depending on the product under development and how the market values that product as function of the development or maturation time.

For the example used in the model of a supply chain of a commodity product, this relationship can be summarized as first having a low value that rises until it reaches a peak of full value at an ideal maturation time, and then declines as it sits in the field either further maturing past its prime or even decaying.

To capture the above dynamics, a table function could be employed but for simplicity consider a trapezoidal relationship between crop value and age (or maturation time). This relationship utilizes four parameters to capture when a crop first has any economic value, the range over which it has full economic value, and the age above which it again has no economic value.

\[
f(t) = \begin{cases} 
0 & t \leq a \\
\frac{1}{b-a} t - \frac{a}{b-a} & a < t \leq b \\
1 & b < t \leq c \\
\frac{1}{c-d} t - \frac{d}{c-d} & c < t \leq d \\
0 & t > d 
\end{cases}
\]  
(6)

Figure S14 provides a visual summary of expression (6)

![Figure S14. Example of Trapezoidal Function Discounting the Value of Crops based on Maturation](image_url)
Note that the expression above assumes a linear change from minimum to maximum value, and constant maximum value between points \( c \) and \( d \). More general trapezoidal shapes are possible that do not necessarily have these features (for example see (Dorp & Kotz, 2003)) and may be more appropriate in specific contexts, but this formulation is sufficient here.

Under the aggregate model framework, which is used in this example supply chain model, the value of the entire stock of work-in-progress inventory is derived by the formulation above. If the vintaging model was used, it would be applied to each cohort of ages.

**A Multinomial Logistic model of Crop Dispositions**

The section immediately below largely restates material in the main article. It is repeated here to allow for a largely self-contained narrative of the development of the methodological comparison model.

In the methodological comparison model, we consider that the producer has three choices to make with respect to units that are actively under development (WIP) 1) Terminate development and move into a finished goods state (for immediate or later sell to the customer), 2) Keep under development to continue to mature (or decay), or 3) Terminate development and destroy or dispose.

From the point of view of a single producer, each of these dispositions are binary (for example a farm cannot simultaneously destroy, harvest, and continue to cultivate a single unit of food). Under a model of a single producer, this economic decision becomes a straight-forward assessment of the expected value of each disposition route (for example weighing the of the costs of shipping and storing goods versus the costs of destroying it, offset by the value that would come from selling if it were sold). However, for a larger model of a system of producers, it is more appropriate to utilize a multinomial logistic model, to represent the probability of a producer choosing any of the above three options.

For some relative economic value \( \pi_i \) for choice \( X_i \), the probability of choosing \( X_i \) is given by the expression below:

\[
P(X_i) = \frac{e^{\beta \pi_i}}{\sum_{i=1}^{N} e^{\beta \pi_i}}
\]  

(7)

In the above, \( \beta \) is the weight the producer places on the concept of economic value. Under a full logistic model that we could fit to observed data, this becomes a free parameter. Here, we have no observed data, but rather a conceptual model. Thus, to simplify the model overall, we can fix values of \( \beta \) to be the inverse of some reference price for the producer (e.g., the price at which a producer sells its goods under normal steady state conditions). This has the advantage of allowing the relative values of each choice, \( \pi_i \),
to be expressed in terms of prices and monetary values, while allowing the expression above to properly reduce to a dimensionless probability.

\[ \beta_i = \frac{1}{\text{reference price}} \forall i \]  

(8)

**Valuing WIP Dispositions**

As discussed above, how the producer derives the relative values of each of the choices is a matter of modeling freedom and should ideally be based in observations of how real produces value these choices. The advantage of the logistic model is that changing these assumptions only changes the relative value of each choice, and thus the relative proportion of the crop delegated to each option, but not the underlying model.

This is the most straight forward valuation in the model and is simply the cost of destroying the units under development. The act of ceasing production and destroying goods is not considered ‘free’ and has a cost assigned to it in the model as an exogenous parameter. This could be expanded by applying a ‘mental resistance’ or ‘sunk cost fallacy price’ to further discourage the disposal of WIP, if evidence supports it. As a note, as modeled here, the value of disposing of goods is always negative. While the other options can be more negative, even if they are strictly positive, some portion of the crop is nevertheless destroyed each period under the multinomial logistic model.

\[ \pi_{\text{destroying WIP}} = - \text{Producer Cost to Dispose of WIP} \]  

(9)

If the producer is to finished development and store the units in the same area (or hold up the same production capacity) while not actually adding value to the good, they would do so under the expectation that they would receive their current expected price for the goods, less the costs for moving into an FG state, less the eventual costs for shipping to the customer.

\[ \pi_{\text{producing}} = \text{Producer Short Run Expected Price} - \text{Cost to move into FG state} - \text{Shipping Costs} \]  

(10)

Combined with the above logistic model, this gives a fraction of the units under development that could be made available, at most, for shipping.

The ‘Production Rate’ flow in the model developed in the main article is based on both the expected future need of goods to fulfill demand from the wholesaler and anticipated spoilage or loss in storage.
Expected Needed Production

\[
\text{Expected Needed Production} = SMOOTH(\text{Anticipated Customer Demand} \\
+ \text{Expected Loss of FG, Time to Update Expected Production Need}) \tag{11}
\]

If the producer were fully willing to meet customer demand and replace any goods previously destroyed or spoiled or other loss in storage, the above alone would move the goods from production starts through to finished goods. However, the value of the units under production and available to be moved into a finished goods state is limited by the logistic model described above. Thus, expression for Production Rate from the main article can be recast into this example context as seen in expression (12) below.

Production Rate

\[
\text{Production Rate} = \min(1, \frac{\text{Maximum Production Rate}}{\text{Expected Needed Production}}) \times \text{Expected Needed Production} \tag{12}
\]

The actual number of units left in the WIP state is ultimately defined by how many units are destroyed and how many units are moved into a finished goods state each period. However, the probability that a producer will choose to destroy, or complete development is also dependent on how the producer values keeping units under development. There are two possible ways to capture the value of leaving work-in-progress alone to continue to age, both of which are explained below:

The first option is both the easiest conceptually, and perhaps the most robust because it introduces the least number of additional assumptions: that keeping the units under development in a WIP state has a null value. In many logistic models, there is a ‘null choice’ or simply a choice of zero value, often used to represent not making a choice at all (e.g., between a red car and a blue car I choose to not buy a car today). For this model, the relative value of holding crops is 0.

Option 1: \( \pi_{holding\ units\ in\ WIP} = 0 \) \tag{13}

The other option is more behaviorally complex, but perhaps more realistic. Here, the producer is assumed to be forward looking, anticipating getting the maximum value from her production that could be expected.

Option 2: \( \pi_{holding\ units\ in\ WIP} = \text{Producer Future Looking Price} - \text{Production Costs} - \text{Shipping Costs} \) \tag{14}
Under this model, the producer is assumed to know the shape of the relationship between age and value discussed above and can expect the maximum fraction of the value of her production if the maturation time is lower than the ideal maturation time, but nothing better than the current value for maturation times higher than the ideal value.

\[
Producer Future Looking Expected Price = \begin{cases} 
\text{Short Run Expected Price} & \text{if } T_{\text{maturation}} < T_{\text{ideal}} \\
\text{Short Run Expected Price} \times \text{Effect of Age on Price} & \text{o.w.}
\end{cases}
\]  
(15)

Ultimately, the choice of option 2, generally, causes the producer to reserve more units in the WIP state each period, as the value of the goods is viewed higher than null.

**Valuing FG Inventory Dispositions**

While the model development immediately above has focused on the valuation and inventory disposition decisions of work-in-progress production, it can be readily applied as well to finished goods inventory in storage as well. Again, the producer has three choices: 1) Make inventory available for the customer, 2) keep finished goods in storage, or 3) dispose of finished goods. As with the work-in-progress inventory, a multinomial logistic function can be used, normalized with \( \beta \) values all chosen to be the inverse of a producer reference price. As a note, the inclusion of this feature has negligible impact on the example system parameterized in the main article but is included for completeness for the reader to experiment with.

As with the previous sector, the value of destroying finished goods can be assumed to be some simple value. It is possible to expand this valuation by considering how destroying inventory frees storage space, but rather than complicate the valuation here, those consideration are rolled into the valuation of holding goods.

\[
\pi_{\text{destroying finished goods}} = -\text{Producer Cost to Destroy Inventory}
\]  
(16)

The value of making inventory available to ship is simply the current spot price, less the cost of shipping those goods. Note that the current spot price is affected by the maturation of the units as described above.

\[
\pi_{\text{Shipping}} = \text{Spot Price} - \text{Shipping Costs}
\]  
(17)

As with the choice to hold WIP to further mature, there are two ways to look at the valuation of holding inventory rather than destroying or shipping it, either with a null value or with a more forward-looking model of valuation.
Option 1: \( \pi_{\text{Hold Inventory}} = 0 \)  

(18)

For the forward-looking estimation, consider the that the opportunity cost of storing an additional unit of goods for an additional unit of time increases with finite storage space, and the only feasible method of storing additional units of goods when storage is full is to acquire additional space at some costs. This is captured in the relationship below:

\[
\text{Cost to Hold Based on Free Space} = \text{Farm Holding Costs} + \frac{\text{Fraction of Storage Full}}{\text{Farm Holding Costs} + \text{Costs to Acquire Storage}} \tag{19}
\]

Furthermore, by holding the finished goods, the producer must be expecting not the current spot price, but some future estimate of the price for their goods. Combined, this gives the following alternative option for valuing holding inventory in this model:

Option 2: \( \lambda_{\text{Hold Inventory}} = \text{Short Run Expected Price} - \text{Cost to Hold Based on Free Space} \)  

(20)

Either of the two options of valuation above presuppose a decision to acquire storage space if full. Thus, we can consider that the producer has a desired total storage space that is approximately equal to the actual finished goods inventory, with perhaps an additional allowance for free space for comfort or other purposes.

\[
\text{Desired Storage Space} = \frac{\text{FG Inventory}}{1 - \text{Producer Desired Fraction Free in Storage}} \tag{21}
\]

The producer will then actively work to adjust the actual storage space to the desired storage space, though perhaps in an asymmetric manner. Specifically, I hypothesize that the producer will be quick to add space but slow to divest of it.

\[
\text{Storage Space} = \text{SMOOTH} \left( \text{Desired Storage Space}, \begin{cases} 
\text{Time to Add to Storage Space} & \text{if Desired Storage Space} > \text{Farm Space} \\
\text{Time to Reduce Storage Space} & \text{otherwise}
\end{cases} \right) \tag{22}
\]
Some Alternative Functional Forms for the Age-Value Relationship

The example in the main article and the development of the formulations above assume a trapezoidal relationship between age of work-in-progress and the value that the producer can extract. As discussed in the main article, multipole alternative shapes could be feasible in different contexts, and even the core trapezoidal shape explored in this article can take on more complex configurations (Dorp & Kotz, 2003).

The methodological comparison model .mdl file that accompanies this article allows for users to experiment with several of the relationships seen in Figure S15.
Null Relationship (Fixed Value)

\[ f(t) = 1 \]

Linear and Saturating Relationship

\[
f(t) = \begin{cases} 
0 & t \leq a \\
\frac{1}{b-a} t - \frac{a}{b-a} & a < t \leq b \\
1 & t > b
\end{cases}
\]

Trapezoidal Relationship

\[
f(t) = \begin{cases} 
0 & t \leq a \\
\frac{1}{b-a} t - \frac{a}{b-a} & a < t \leq b \\
1 & b < t \leq c \\
\frac{1}{c-d} t - \frac{d}{c-d} & c < t \leq d \\
0 & t > d
\end{cases}
\]

Asymmetric Gaussian Relationship

\[
f(t) = e^{-\frac{(t - T_{\text{ideal}})^2}{2\sigma^2}} \\
\sigma = \sigma_{\text{ideal}} \quad \text{if } t < T_{\text{ideal}} \\
\sigma = \sigma_u \quad \text{o.w.}
\]

Figure S15. Examples of Price-Value Relationships
Parameterization of the Methodological Comparison Model

The detailed parameterization of the example supply chain model used in the main article is largely omitted for the sake of space. This was done also because the focus of that section of the article was not about the influences of specific parameter choices, but rather to illustrate how the methodology introduced can generate fundamentally different modes of behavior for otherwise identically parameterized models. The .mdl file included with the article comes parameterized in the same manner as used for the main article, but those parameter values are explicitly listed below as well.

Below are the values used for the age-value relationship, which for the article utilized the trapezoidal relationship, which is described in expression (6) above.

<table>
<thead>
<tr>
<th>Parameter Name</th>
<th>Description or Note</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Minimum age of any value in the Trapezoidal relationship</td>
<td>4 weeks</td>
</tr>
<tr>
<td>B</td>
<td>Minimum age of full value in the Trapezoidal relationship</td>
<td>10 weeks</td>
</tr>
<tr>
<td>C</td>
<td>Maximum age of full value in the Trapezoidal relationship</td>
<td>14 weeks</td>
</tr>
<tr>
<td>D</td>
<td>Maximum age of any value in the Trapezoidal relationship</td>
<td>30 weeks</td>
</tr>
<tr>
<td>Initial average WIP Age</td>
<td>To ensure steady state at the initialization of the model, this value should be within (inclusive) of B and C above. Chosen to be the average of those two numbers for simplicity of exposition</td>
<td>12 weeks</td>
</tr>
</tbody>
</table>

The structure used to track the average age of WIP inventory utilized in this example model is most similar to the ‘Coflow with Experience’ structure discussed in detail in *Molecules of Structure* (Hines, 2005), though the unit balancing takes on a different form to be dimensionally consistent with the rest of the system. For completeness of exposition, a diagram of just this portion of the model is repeated in Figure S16. Note that any structure that captures the development time of the cohort of interest could be used to relate this time to the economic value that could be extracted from the goods.
As stated in the main article in the formulation for the co-flow that monitors the average age of work-in-progress inventory, the ‘Average Age of New Production Starts’ may have a value of 0 units of time, or some other non-negative value. For example, when applied to employee experience in a firm, an employee may arrive with some pre-existing experience. However, in the context of a food producer planting crops it may be safe to assume that newly planted crops arrive with no pre-existing maturation. The model is flexible to allow for this assumption to be relaxed based on specific circumstance (for example, buying partially matured nut trees or fully matured sows rather than starting from seeds or piglets).

Furthermore, while it may be generally safe to assume that the ‘Rate of Age Gain’ is constant and unitary (i.e., 1 weeks/week or 1 years/year or similar). The formulation itself does allow for some flexibility if, as an example, a fertilizer was applied to speed maturation, or a drought hit and slows maturation down.

In general, the structure here is most similar to the ‘Coflow with Experience’ structure discussed in detail in *Molecules of Structure* (Hines, 2005), though the unit balancing takes on a different form to be dimensionally consistent with the rest of the system. Note that any structure that captures the development
time of the cohort of interest could be used to relate this time to the economic value that could be extracted from the goods.

Table S2. Parameterization for the Co-Flow Structure Monitoring Average WIP Age

<table>
<thead>
<tr>
<th>Parameter Name</th>
<th>Description or Note</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate of Maturation Gain</td>
<td>Rate at which unit under development gains age.</td>
<td>1 Weeks/Week</td>
</tr>
<tr>
<td>Average Age of New Units</td>
<td>The average maturation of brand-new production starts.</td>
<td>0 Weeks</td>
</tr>
</tbody>
</table>

Throughout the model there are time constants that affect the rate at which entities in this model supply chain either incorporate information and update estimations or limit the rate at which they can perform actions. These values were chosen to be behaviorally realistic (for example the producer incorporating price changes into their forward projection affecting production starts much more quickly than the customer adjusting their demand in response to those same price fluctuations), but again the primary purpose of this model is not to explore the sensitivity to these parameters but rather illustrate that different modes of behavior emerge when utilizing the methodological contributions illustrated in the article.

Table S3. Parameterization for Time Constants

<table>
<thead>
<tr>
<th>Parameter Name</th>
<th>Description or Note</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time to adjust Production Schedule</td>
<td>Average time to adjust the actual unit production starts (or planting schedule) to the desired value.</td>
<td>26 Weeks</td>
</tr>
<tr>
<td>Time to adjust expected mixed variable costs</td>
<td>Time, on average, for the producer to update its expectation of the mixed variable costs it will typically incur per unit production started.</td>
<td>2 Weeks</td>
</tr>
<tr>
<td>Producer Processing Time</td>
<td>The desired typical time it takes for the producer to process WIP goods, either by disposing of them or moving them along into finished goods inventory.</td>
<td>4 Weeks</td>
</tr>
<tr>
<td>Producer finished goods disposal time</td>
<td>Time, on average, for the producer to dispose of stock of finished goods awaiting shipment to the customer</td>
<td>4 Weeks</td>
</tr>
<tr>
<td>Time for customer to adjust demand</td>
<td>The time, on average, for the demand from the Customer to change based on the indicated demand</td>
<td>4 Weeks</td>
</tr>
<tr>
<td>Time for producer to adjust short-run expected price</td>
<td>Time, on average, for the producer to incorporate the spot price into its expected price</td>
<td>1 Week</td>
</tr>
<tr>
<td>Time for customer to adjust short-run expected price</td>
<td>Time, on average, for the customer to incorporate the producer spot price into its expected price</td>
<td>24 Weeks</td>
</tr>
<tr>
<td>Average Shelf Life of Producer Stored FG</td>
<td>The average time the unit that has been moved to a finished goods state, but is still being stored at the producer, can sit in storage before spoiling and being disposed of. For non-foodstuffs this could be an average obsolescence time</td>
<td>24 Weeks</td>
</tr>
</tbody>
</table>
The parameters below were used to form the price formation mechanism at use in this model and as described in the Effect of Inventory Coverage on Price and Effect of Spot Prices on Demand sections of this appendix.

**Table S4. Parameterization for the Effects of Inventory Coverage and Elasticities**

<table>
<thead>
<tr>
<th>Parameter Name</th>
<th>Description or Note</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Producer Desired Inventory Coverage</td>
<td>Weeks supply of inventory the producer wants to have on hand</td>
<td>2 Weeks</td>
</tr>
<tr>
<td>Sensitivity of Producer Price to Producer Inventory Coverage</td>
<td>Factor affecting how 'steep' the inverse relationship between inventory coverage and price is. Note that based on this formulation, this is assumed a positive value here for the expected inverse relationship. Higher positive values of this factor imply more sensitivity.</td>
<td>2 (dimensionless)</td>
</tr>
<tr>
<td>Elasticity of Customer Demand</td>
<td>Under an assumption of a linear demand curve near the reference prices and demand, this is the negative value of the slope of that curve. Note that this parameter is assumed to take on a positive value under the default assumptions of decreasing demand with increasing spot prices. High positive values of the factor create a steeper, but still negatively sloped, demand curve.</td>
<td>1 (dimensionless)</td>
</tr>
</tbody>
</table>

As described in the main article and partially restated in this appendix, the multinomial logistic choice model ultimately depends on the relative difference in perceived value of each disposition option. Thus, each option must have some manner by which that value can be calculated. For this specific model, this value is simply determined by comparing the expected profit from each disposition route along the supply chain. The costs and baseline revenue values used are given below.
<table>
<thead>
<tr>
<th>Parameter Name</th>
<th>Description or Note</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference Producer Price</td>
<td>Price at which the Producer experiences its reference Gross Margin and Reference Planting. Sets the default profitability expectations in steady state for the producer</td>
<td>$1.1/unit</td>
</tr>
<tr>
<td>Producer Raw Material Costs</td>
<td>Raw cost per unit (i.e., the variable cost) the producer endures</td>
<td>$0.05/unit</td>
</tr>
<tr>
<td>WIP Development Costs</td>
<td>Cost of developing a single unit of productions starts for a single unit of time that the producer endures</td>
<td>$0.01/unit/week</td>
</tr>
<tr>
<td>Producer Cost to Dispose of or Abandon WIP</td>
<td>The cost incurred by the producer to dispose of a unit being actively developed in a WIP state. Note that this could not only be the actual material cost (labor and equipment) but also could be extended to include physiological costs from sunk cost fallacy or similar resistance to disposing units that have already had resources invested in their development.</td>
<td>$2/unit</td>
</tr>
<tr>
<td>Producer Cost to Place in FG State</td>
<td>Costs, per unit, that the producer incurs to move a unit from the WIP to the FG state.</td>
<td>$0.1/unit</td>
</tr>
<tr>
<td>Producer Shipping Cost</td>
<td>Costs, per unit, that the producer incurs to process and ship goods for the Customer.</td>
<td>$0.1/unit</td>
</tr>
<tr>
<td>Producer Cost to Dispose of FG Inventory</td>
<td>The cost incurred by the producer to dispose of a unit that is being stored after production and before shipping to the customer. Note that this could not only be the actual material cost (labor and equipment) but also could be extended to include physiological costs from sunk cost fallacy or similar resistance to disposing units that have already had resources invested in their development and storage.</td>
<td>$1/unit</td>
</tr>
</tbody>
</table>
References to the Appendix


